LAGRANGIAN APPROACH
TO THE MECHANICS (Acoustics) OF
PENETRABLE POROUS MATERIALS
Sergey LOPATNIKOV

Mentis Sciences, Inc.
215 Canal Street
Manchester, NH 03101

www.mentissciences.com
603-624-9197
Advanced Materials for DoD Applications
SERGEY LOPATNIKOV’s Background

1977 - Ph.D in Physics and Mathematics, Before 2000, Head of Branch of mathematical modeling for All Union Institute of Nuclear Geophysics and Geochemistry (Moscow, USSR), Lead scientist for Chemical Department of Moscow State University. Since 2000 Distinguished Visiting Professor, Researcher for Composite material Center of the University of Delaware, since 2014 – Principal scientist for Mentis Sciences, Inc.

Plasma and non-linear wave propagation: self-focusing instability of strong EM waves in plasma with magnetic field; self-focusing of strong EM in media with non-local non-linearity; non-linear excitation of ionospheric waveguide

Soft matter: interaction of macro-molecules (nano-particles) embedded in nematic and smectic Liquid Crystals; Key&Lock interaction of such particles; Chemical switch of orientation of thin layers of LQ; Shear acoustic wind acting of particles in LC; First high-strain-rate measurements of properties of Shear Thickening Fluids; Novel representation of SHPB data

Chemistry & quantum mechanics: resonant type of manipulation of mono-molecular gas-phase reactions; Non-equilibrium noise of Scanning Tunneling Microscope; Contracted mapping method of solution of symmetric eighenvalue problem, particularly Hartrey-Fock equation; Mechanics of Carbon Nano-Tubes

Material Sciences: Theory of metal foam deformation and their applications; Nonlinear model of VARTM, New active materials: foams, infiltrated with magnetic fluid; Theory of conformable robots; Correction of Kolsky’s theory of SHPB; Method of creation of short hypersonic jet for rain erosion stability of materials (up to 1200 m/sec).

Geophysics: Method of tsunami warning (USSR patent); Maximum of Information approach to optimization of Side-Scan Sonar signal for mapping of diffuse distributed targets; Acoustic heating of near-borehole zone for it cleanup from contaminants; Novel type of acoustic source for well logging. Novel mechanisms of primary and secondary migration of hydrocarbons and origination of oil and gas deposits;
OUTLINE

1. Brief History. Frenkel, Biot and others.
2. Why poroelastics.
3. Why existing theories of poroelastics, including M.Biot’s theory has to be reconsidered? – Requirement to the theory.
4. Reminiscence of Lagrangian approach.
5. Langrangian approach in application to fluid-filled poroelastics
   - External and internal variables and generalized Cauchy-Born hypotheses;
   - Density of Lagrangian and variations with restrictions;
5. Physical meaning of governing equation of poro-mechanics and relations to the composite theory.
6. Natural boundary conditions.
7. Example of investigation of stationary state filled with Van der-Walls fluid.
8. Linear waves in poroelastics.
9. Interaction of waves in random porous media.
10. Two possible new applications of the theory in medicine.
10. Conclusions.
WHY POROUS MATERIALS?

GEOLOGY AND GEOPHYSICS
- Reservoirs
- Seismology, Logging
- Marine sediments
- Side-Scan Sonars...

TECHNOLOGY
- Explosions...
- VARTM...
- Turbulence control

BIOLOGY & MEDICINE
- Sponge
- Bone
- Lung tissue
SPECIFIC OF PENETRABLE POROUS MATERIALS

REGULAR SOLID BODY
Each point of solid body is mapping into “new position”. State of each point is characterizing by three components of displacements of each material point.

POROUS BODY WITH CLOSED PORES
Each macro-point of such body is mapping into “new position of macro-point”. State of each point is characterizing by three components of displacements of each material macro-point. No macro-motion of fluid.

PENETRABLE POROUS BODY
Each macro-point point of matrix is mapping into it “new matrix macro-point position”. However, there is possible macro-motion of fluid. Thus, state of each point is characterizing by 3 components of displacements of matrix macro-point and 3 displacements of fluid.
HENRY DARCY’S LAW

Darcy establishes proportionality between flow rate of a fluid and through homogeneous penetrable porous medium to gradient of pressure:

\[ Q = -\frac{kA(p_b - p_a)}{\mu L} \]

The point is that applications of Darcy law is limited. Basically, in spite of there is flow of fluid, Darcy considered particular case of stationary state of filtration, of incompressible fluid through incompressible matrix.

It does not provide any information about transient regimes of setting up this stationary state.

Definitely, these transient regimes are important and more advanced theory must be developed. It became especially actual in the first part of XX-th century when in geophysics seismic methods start to develop.

Particularly, one has to speak about acoustic of porous materials
BRIEF HISTORY

A. Ivanov, 1939

SEISMOELECTRIC EFFECT

In 1944 Frenkel understood that effect is related with relative motion of electrolyte and matrix, built appropriate theory.

As a side effect, he invented two types of longitudinal waves and investigated their attenuation in Darcy limit of fluid filtration. The problem: only low frequencies. Conclusion is that wave of second type is not of interest for considering problem (seismo-electric effect).

Prof. V. Schelkachev

Independently, the low-frequency second wave under the name of piezo-conductivity wave has been investigated in 1948 by Russian Petroleum geophysist Prof. V. Schelkachev.

In his monography “Elastic regime of water-pressure reservoir systems”, based on the Darcy law, he introduced coefficient of reservoir piezo-elasticity and proposed diffusion type of equation for piezo-conductivity wave.

\[
\frac{\partial p}{\partial t} = D_p \Delta p
\]

\[
D_p = \frac{k}{\nu \left( \frac{\phi}{\rho_f c_f^2} + \frac{1}{\rho_m c_m^2} \right)}
\]
WHY WE NEED BETTER THEORY

1. **Middle school problem:** One put some amount of fluid of mass $M$ with known equation of state into rigid volume $V$. The question is: in which state will be a fluid and what pressure will act at the walls of volume.

$$\rho_f = \frac{M}{V}$$

$$p_f = p_f(\rho_f, T)$$

2. **The problem not for middle school:** Similar problem, but $V$ – is a volume of porous space of compressible porous material.

**IT IS IMPOSSIBLE TO SOLVE THIS PROBLEM IN FRAME OF BIOT’S THEORY. WHY?**

$$\iiint_{\Omega} \delta W d\Omega = \iiint_{\Omega} \left( \tau_{xx} \delta \varepsilon_x + \tau_{yy} \delta \varepsilon_y + \tau_{zz} \delta \varepsilon_z + \tau_{yz} \delta \varepsilon_x \delta \varepsilon_y + \tau_{zx} \delta \varepsilon_x \delta \varepsilon_z + \tau_{xy} \delta \varepsilon_y \delta \varepsilon_z + p \delta \zeta \right) d\Omega$$

$$\text{div} \mathbf{u} = \text{div} \mathbf{w} = 0 \Rightarrow \varphi = \varphi_0$$
MAURICE A BIOT


Biot introduced empirical elastic coefficients characterizing the mechanical properties of fluid filled penetrable poroelastics and proposed two simple gedanken experiments for their measurements.

Also, he analyzed the attenuation of longitudinal waves for higher frequencies more deeply. He generalizing Darcy low for so fast motions that the length of viscous wave becomes comparable of shorter that characteristic size of material pores (capillaries). The characteristic frequency $\Omega_B$ under which the length of viscous wave becomes comparable with characteristic size of material pores are usually referred as “Biot’s Frequency” after M/ Biot.

OTHER IMPORTANT NAMES

Rakmatullin, Gertsma, Nikolaevskiy, Bedford and Drumheller, Berryman and Thigpen, de Boer, Sanches-Palencia, Wilmanski, A. Cheng, etc. Most of the works are aimed to provide more solid background for Biot’s approach and generalize Biot’s approach to non-linear case.
• Biot’s “energy” – also is not actually the energy. In absence of long-range forces, the real energy is additive. Biot’s energy is not.

• What is the nature of adjacent mass, introduced by M.Biot? Which boundary conditions one are appropriate, etc?

• There is no clear transition to some practically important theories, for example, to the theory of liquids with gas bubbles.

• All known attempts to build “exact methods” of averaging of micro-dynamic equations lead to appearance in the equation of fluid dynamic equations of the force \( \nabla \phi p_f \) instead of \( \phi \nabla p_f \). Prof. V. Nikolaevskiy from Russian Institute of The Physics of Solid Earth was first, who recognized the problem and introduced the artificial force: \( F_N = -p_f \nabla \phi \), which compensate the term with porosity gradient: \( \nabla \phi p_f = \phi \nabla p_f + p_f \nabla \phi \). Without Nikolaevskiy’s force, one can neglect with marked term only if characteristic length of porosity change is very big in comparison with the length of a perturbation, (wave length).
REQUIREMENT TO THE THEORY

**CONSISTENCY**
Theory of poroelastics must be internally consistent and match to general laws of physics.

**UNIVERSALITY**
Theory of poroelastics must be applicable for all types of poroelastic material, for static and dynamic, linear and nonlinear processes.

**OPENNESS**
Theory of poroelastics must provide clear and natural way for incorporation of interaction of physical fields like (electromagnetic, gravity, etc) and other physical processes.

Example: Conformable Robots as distributed media

From macro-point of view, one can consider a Conformable Robot as a continuum

From micro-point of view, one can consider each point of such continuum as a discrete machine
LAGRANGIAN ALGORITHM

IDENTIFY PARAMETERS OF STATE

TO BUILT LAGRANGIAN: \[ L = T - U \]

DEFINE ACTION AND MINIMIZE IT:

\[ \delta A = \delta \int_{t_1}^{t_2} L(q, \dot{q}) \, dt = 0 \]

GET EULER’S GOVERNING EQUATIONS

FIELD THEORY

\[ L(q, \dot{q}) = \int_{\Omega} l \left( u_i(x_k), \frac{\partial u_i(x_k)}{\partial x_k} \right) d^3x_k \, dt \]

\[ A = \int_{t_1}^{t_2} \int_{\Omega} l \left( u_i(x_k), \frac{\partial u_i(x_k)}{\partial x_k} \right) d^3x_k \, dt \]

\[ \delta A = \delta \int_{t_1}^{t_2} L(q, \dot{q}) \, dt = 0 \]
EXTERNAL PARAMETERS

External parameters – local parameters characterizing the deformation of porous manifold as a whole.

Examples: metric tensor $g_{ij}(X)$ deformation gradient $\Gamma = \left[ \frac{\partial \xi_i}{\partial X^j} \right]$ strain tensor; strain-rate tensor, etc.

INTERNAL PARAMETERS CHARACTERIZE THE STATE OF REV

Internal parameters are fields of parameters of state “of the points of manifolds” describing local state of porous materials.
SIMPLE MODEL

MODEL ACTION

\[ A = \int_{\Omega} (1 - \varphi) \left( k_S - \varepsilon_S \left( \eta^\alpha \right) \right) d \left( (1 - \varphi) \rho_S \Omega_S \right) + \int_{\Omega} \varphi \left( k_f - \varepsilon_f \left( \rho_f \right) \right) d \left( \varphi \rho_f \Omega_f \right) \]

AVERAGING OF KINETIC ENERGY

\[ 2k_f \left( \mathbf{X} \right) = \left\langle \rho_f \mathbf{v}_f^2 \right\rangle \approx \rho_f \left\langle \mathbf{v}_f \right\rangle^2 + \rho_f \left\langle \left( \mathbf{v}_f - \left\langle \mathbf{v}_f \right\rangle \right)^2 \right\rangle \]

FLUCTUATIONS ARE PROPORTIONAL TO AVERAGE SPEED OF FLOW

\[ \left| \mathbf{V}_f - \left\langle \mathbf{V}_f \right\rangle \right| = C \cdot \left\langle \mathbf{V}_f \right\rangle \quad \Rightarrow \quad k_f = \frac{1}{2} \rho_f \left\langle \mathbf{v}_f \right\rangle^2 + \rho_{12} \cdot \left( \left\langle \mathbf{v}_f \right\rangle - \left\langle \mathbf{v}_s \right\rangle \right)^2 \]

EXACT ESTIMATES

2D- periodic set of cylinders

\[ \rho_{12} \approx \varphi \left( 1 - \varphi \right) \rho_f \]

3D-periodic set of spheres

\[ \rho_{12} \approx 0.5 \varphi \left( 1 - \varphi \right) \rho_f \]

Similar consideration is possible for solid matrix. Thus, added mass is separated in fluid and solid parts.
BONDS

RELATIONSHIP BETWEEN INTERNAL AND EXTERNAL VARIABLES: INCREMENTAL APPROACH

\[ \eta^\alpha - \text{are internal variables} \Rightarrow \delta \eta^\alpha = \hat{L}(\eta^\nu) \delta \Gamma^\beta \]

\[ \Gamma^\beta - \text{are external variables} \]

\[ \frac{\delta \Delta \Omega}{\Delta \Omega(X)} = \nabla \cdot \delta \mathbf{u}_s \equiv \delta \varepsilon_{ii} \]

DIFFERENTIAL FORM OF CONTINUITY EQUATION

\[ -\frac{\delta \phi}{1-\phi} - \delta \varepsilon_S + \partial_i \xi_S^i = 0 \]

\[ \frac{\delta \phi}{\phi} + \frac{1}{\phi} \left( \delta \xi_f^i - \delta \xi_S^i \right) \cdot \frac{\partial \phi}{\partial X_i} - \delta \varepsilon_f + \partial_i \delta \xi_f^i = 0 \]

SHEAR COMPONENTS

\[ \{ \delta \varepsilon_{ij} \} = L_{ijkl} \{ \varepsilon_{ij} \} \{ \delta \varepsilon_{ij} \} \]

SAMPLE

\[ \{ \varepsilon_{ij} \} = C \{ \varepsilon_{ij} \}; \]

\[ \{ \varepsilon_{ij} \}_\omega = i \omega \eta \{ \varepsilon_{ij} \}_\omega \]
Governing Euler’s equations

**MASS CONSERVATION FOR SOLID AND FLUID**

\[
\frac{\partial (1 - \varphi) \rho_S}{\partial t} + \text{div}(1 - \varphi) \rho_S \mathbf{v}_S = 0
\]

**MOMENTUM CONSERVATION FOR SOLID AND FLUID**

\[
(1 - \varphi) \rho_S \left( \frac{\partial \mathbf{v}_{S_i}}{\partial t} + (\mathbf{v}_S \cdot \nabla) \mathbf{v}_{S_i} \right) - \nabla_j \left( \Sigma_{ij} \left( \overline{\mathbf{E}}_S, \dot{\varphi}, \dot{\varphi}, \ddot{\varphi} \right) \right) + p_f \nabla \varphi + \hat{\mathbf{b}}(t)(\mathbf{v}_S - \mathbf{v}_S) = 0
\]

\[
\varphi \rho_f \left( \frac{\partial \mathbf{v}_{f_i}}{\partial t} + (\mathbf{v}_f \cdot \nabla) \mathbf{v}_{f_i} \right) + \nabla \varphi p_f \left( \rho_f, \dot{\rho}_f, \dot{\varphi}, \ddot{\varphi} \right) - p_{f1} \nabla \varphi + \hat{\mathbf{b}}(t)(\mathbf{v}_f - \mathbf{v}_S) = 0
\]

**POROSITY DYNAMICS EQUATION**

\[
p_f - p_s - (1 - \varphi) \rho_S \frac{\partial e_s}{\partial \varphi} = F_{\varphi}
\]

\[
F_{\varphi} = \rho_S l_{s\varphi}^2 \left[ (1 + \lambda_{\varphi} \varphi) \frac{d^2 \varphi}{dt^2} + \lambda_{\varphi} \left( \frac{d \varphi}{dt} \right)^2 + \eta \frac{d \varphi}{dt} \right]
\]

**SHEAR DYNAMICS**

\[
\frac{d \overline{\mathbf{E}}_{ij}}{dt} = L_{ijkl} \frac{d \varepsilon^{kl}}{dt}
\]

\[
p_f = -\rho_f^2 \frac{\partial e_f (\rho_f)}{\partial \rho_f}
\]

\[
\Sigma_{ij} = (1 - \varphi) \rho_S \left( \frac{1}{3} \frac{\partial e_s}{\partial \overline{\varepsilon}_{nm}} \delta_{nm} - \frac{1}{3} \frac{\partial e_s}{\partial \overline{\varepsilon}_{ij}} L_{nmij} \delta_{mn} \right) \delta_{ij} + \frac{\partial e_s}{\partial \overline{\varepsilon}_{ij}} L_{klij}
\]
EXAMPLE OF STATIONARY STATE: VAN DER WAALS FLUID

BOUNDARY CONDITIONS

EQUATION OF STATE FOR MATRIX

\[ e_s = e_{s0} + \frac{1}{2} \lambda_s I_1^2 + \frac{1}{2} \mu^* I_2 + K_{\varphi} I_1 \Delta \varphi + \frac{1}{2} K_{\varphi} \Delta \varphi^2 \]

VAN DER WAALS EQUATION OF STATE

\[
\left( p_f + \left( \frac{M_f}{\mu_f} \right)^2 \frac{a}{\Omega_f^2} \right) \left( \Omega_f - \left( \frac{M_f}{\mu_f} \right) b \right) = \frac{M_f}{\mu_f} RT
\]

PRESSURE SHIFT IN COMPARISON WITH INCOMPRESSIBLE MATRIX

\[
\Delta p_f = -(1 - \varphi_{00}) \left( \frac{M_f}{\mu_f \varphi_0} \frac{RT}{(\varphi_{00} - \frac{M_f}{\mu_f \varphi_0}) b} - \left( \frac{M_f}{\mu_f \varphi_0} \right)^2 \right) \left( \frac{M_f}{\mu_f \varphi_0} \frac{\varphi_{00} RT}{(\varphi_{00} \Omega - \frac{M_f}{\mu_f \varphi_0}) b} - 2 \left( \frac{M_f}{\mu_f \varphi_0} \right)^2 \right)
\]

Porosity equilibrium equation

\[ p_f - p_s - (1 - \varphi) \rho_s \frac{\partial e_s}{\partial \varphi} = 0 \]
LINEAR THEORY AND ACOUSTIC WAVES

IDENTIFY EQUILIBRIUM STATE OF THE POROUS MATERIAL FILLED WITH FLUID

PRESENT INTERNAL VARIABLES AS A SUM OF STATIONARY SOLUTION PLUS PERTURBATIONS:

\[ \eta_\alpha = \eta_{\alpha \text{eq}} + \tilde{\eta}_\alpha \rightarrow \tilde{\eta}_\alpha \sim \text{small} \]

SUBSTITUTE EXPRESSION IN GENERAL GOVERNING EQUATION AND HOLD ONLY FIRST ORDER OF PERTURBATION

LINEARIZE BOUNDARY CONDITIONS

\[ \hat{M} \left( \eta_{\beta \text{eq}}(X), t \right) \tilde{\eta}_\alpha = 0 \]

Parameters of linearized theory are:

- \( \frac{\partial e_s}{\partial e_0 \partial e_{mn}} \)
- \( \frac{\partial e_s}{\partial e_0 \partial \varphi} \)
- \( \frac{\partial^2 e_s}{\partial \varphi^2} \)
- \( L_{ijkl} \)
- \( \varphi_0 \)
- \( \rho_{S0} \)
- \( \rho_{f0} \)
- \( \sigma_{ij0} \)
- \( p_{f0} \)
- \( c_{f0} \)
DISPERSION EQUATION FOR WAVES

TWO FORMS OF DISPERSION EQUATIONS

\[ k = k(\omega) \quad \omega \ - \ real \quad A \ CURVE \ IN \ 6-D \ SPACE \]

\[ \nabla^2 A_1\omega + k_1^2(\omega) A_1\omega = 0 \quad - \ WAVE \ OF \ THE \ FIRST \ KIND \]

\[ \nabla^2 A_2\omega + k_2^2(\omega) A_2\omega = 0 \quad - \ WAVE \ OF \ THE \ SECOND \ KIND \]

\[ \nabla^2 \Psi_\omega + k_S^2(\omega) \Psi_\omega = 0; \quad \nabla \cdot \Psi_\omega = 0 \quad - \ SHEAR \ WAVE \]

\[ u_S = \nabla \left( e_{s1}(\omega) A_1\omega + e_{s2}(\omega) A_2\omega \right) + a_S(\omega) \nabla \times \Psi_\omega \]

\[ u_f = \nabla \left( e_{f1}(\omega) A_1\omega + e_{f2}(\omega) A_2\omega \right) + a_f(\omega) \nabla \times \Psi_\omega \]

One immediately has Green’s functions for all waves and presentation of source!

\[ \omega = \omega(k) \quad k \ - \ real \quad TWO \ SERFACES \ IN \ 3-D \ SPACE: \quad \Re \omega(k) \quad \Im \omega(k) \]

\[ V_f(k) = \frac{\omega(k)}{|k|} \cdot \frac{k}{|k|} \quad \rightarrow \quad V_g = \nabla_k \omega(k) \]
Dispersion equation in the form: \( \omega = \omega(k) \)

Porosity dynamics with account of porosity inertia without dissipation is incorporated in \( k = k(\omega) \) representation

**a)** Below Biot’s frequency, the real part of wave number of wave of second type is exactly equal to ZERO. We will consider the meaning of this fact below.

**b)** There is resonance associated with porosity dynamics.

**OBSERVATIONS:**

- **Pore Resonance**
- **BIOT’S FREQUENCY**
- \( \text{Re} \omega \equiv 0 \)
LOW FREQUENCIES

THE FIRST WAVE: \[ \omega \approx kc_1 + iDk^2 \]
\[ \partial_x A_1 - c_1 \partial_t A_1 = D \Delta A_1 \]
\[ \eta = x + c_1 t; \quad \tau = t \]
\[ \partial_\tau A_1 = D \Delta_\eta A_1 \]

THE SECOND WAVE: \[ \omega \approx iD_p k^2 \]
\[ \partial_t A_2 = D_p \Delta A_f \]

\[ D \sim D_p = \frac{k}{\nu \left( \frac{\varphi}{\rho_f c_f^2} + \frac{1 - \varphi}{\rho_m c_m^2} \right)} \]

It is just the same equation for the wave of piezo-conductivity, investigated by V. Schelkachev in 1948!

THE PHYSICAL MEANING OF BIOT FREQUENCY AND CGARACTER OF SECOND WAVE

Oscillator with attenuation:
\[ \beta = \gamma \pm \sqrt{\gamma^2 - \omega_0^2} \]
\[ y(t) = A \cdot e^{-\beta_it} \]
\[ y(t) = A \cdot e^{-\gamma t} \sin(\omega_f t + \varphi) \]

There is exact analogy between Biot's frequency and critical frequency of oscillator with attenuation.
In homogeneous media the right parts are equal to zero

\[ \Delta A_1 + k_1^2 (\omega) A_1 = \hat{q}_{L11} (\omega, X) A_1 + \hat{q}_{L12} (\omega, X) A_2 + \hat{Q}_{L11} (\omega, X) \Psi_1 + \hat{Q}_{L12} (\omega, X) \Psi_2 \]
\[ \Delta A_2 + k_2^2 (\omega) A_2 = \hat{q}_{L21} (\omega, X) A_1 + \hat{q}_{L22} (\omega, X) A_2 + \hat{Q}_{L21} (\omega, X) \Psi_1 + \hat{Q}_{L22} (\omega, X) \Psi_2 \]
\[ \Delta \Psi_1 + k_S^2 (\omega) \Psi_1 = \hat{q}_{S11} (\omega, X) A_1 + \hat{q}_{S12} (\omega, X) A_2 + \hat{Q}_{S11} (\omega, X) \Psi_1 + \hat{Q}_{S12} (\omega, X) \Psi_2 \]
\[ \Delta \Psi_2 + k_S^2 (\omega) \Psi_2 = \hat{q}_{S21} (\omega, X) A_1 + \hat{q}_{S22} (\omega, X) A_2 + \hat{Q}_{S21} (\omega, X) \Psi_1 + \hat{Q}_{S22} (\omega, X) \Psi_2 \]

Usually, \(|k_2| >> |k_1|\) \quad \text{In consolidated materials: } |k_1| \sim |k_S|

As a result, for scattering of first wave into second wave 1-D conditions can be significantly simplified
DEPENDENCE OF ATTENUATION FROM FLUID VISCOSITY

How to check? - To fill material with glycerin and change temperature!

**EXPERIMENT**

\[ \gamma \propto \nu^{-1} - \text{Biot} \]

\[ \gamma \propto \nu^{1/2} - \text{Our Theory} \]
The major idea is generation of radio-pulses of pressure of predefined amplitude and relatively low frequencies. Propagating through tissues, these pulses will change the state and elastic parameters of tissue due to their non-linearity. As a result, harmonics will be generated. Filtering these harmonics, one can get the map of a new parameter: coefficient of non-linearity of tissues.
1. We presented open for the developments general theory of fluid-filled porous materials based on Lagrangian approach.
2. In frame of presented theory one can consistently find equilibrium state of porous material.
3. Linearized dynamic equations in the neighborhood equilibrium state consistently follow from the theory.
4. We showed that theory naturally includes attenuation related with each introduced degree of freedom.

Some other results, which are not presented in the lecture

1. Theory of penetrable porous materials, filled with ferro-fluid.
2. Theory of propagation of waves in porous materials filled with fluid with gas bubbles, distributed over sizes.
4. Theory of “conformable robots”.