Efficient Computation of Belief Theoretic Conditionals for Time Sensitive Uncertainty Reasoning Applications

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Fig. 1: DS-Conditional-One model. Quantities related to arbitrary $B(A)$ (or $B(A)$) computation when $A = \{a_1, a_2, ..., a_m\}$, $A = \{a_0, a_1, ..., a_m\}$, and $B = \{a_2, a_3\} \subseteq A$.

Fig. 2: DS-Conditional-All model. Quantities related to $B(B|A)$ (or $B(B|A)$) computation for all $B \subseteq A$ when $A = \{a_1, a_2, ..., a_m\}$, and $A = \{a_0, a_1, ..., a_m\}$.

Fig. 3: Computational and space complexity comparison of DS-Conditional-One (or DS-Conditional-All) model with the specialization matrix approach. (Theoretical computational times are calculated assuming 10,000,000 computational iterations per second.)

The following notation is required for our work[1, 2]:

$$S(A; B) = \sum_{\Theta \subseteq A} m_{C \cup D}.$$  \hspace{3cm} (1)

The following alternate expression is useful for the Dempster’s conditional:

$$\Delta(A; B) = \sum_{\Theta \subseteq A} m_{C \cup D}.$$  \hspace{3cm} (2)

The following alternate expression is required for our work[1, 2]:

$$\Delta(A) = \sum_{\Theta \subseteq A} m_{C \cup D}.$$  \hspace{3cm} (3)

We need the following alternate expression for the FH conditional:

**Proposition 1** Consider the BoE $\mathcal{E} = \{\Theta, \bar{\Theta}, m_{\mathcal{E}}\}$ and $A \subseteq \bar{\Theta}$. Then, we may express $B(\bar{\Theta}|A)$ as

$$B(\bar{\Theta}|A) = B(\bar{\Theta} A) - S(\bar{\Theta} A | B) \cup \Theta, \quad B \subseteq \Theta.$$  \hspace{3cm} (4)

The following alternate expression is useful for the Dempster’s conditional:

**Proposition 2** Consider the BoE $\mathcal{E} = \{\Theta, \bar{\Theta}, m_{\mathcal{E}}\}$ and $A \subseteq \Theta$ s.t. $B(\bar{\Theta}) \neq 1$. Then, $B(\bar{\Theta}|A)$ can be expressed as

$$B(\bar{\Theta}|A) = B(\bar{\Theta} A) + S(\bar{\Theta} A | B) \cup \Theta, \quad B \subseteq \Theta.$$  \hspace{3cm} (5)

Dempster’s and FH Conditional Beliefs of All Propositions:

$$\mathcal{B}(A) = \mathcal{B}(A) \cup \Theta, \quad B \subseteq \Theta.$$  \hspace{3cm} (6)

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$$\mathcal{B}(A) = \mathcal{B}(A) \cup \Theta, \quad B \subseteq \Theta.$$  \hspace{3cm} (7)

To Compute

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<th>Dempster’s and FH Conditional Beliefs of All Propositions:</th>
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Comparison with Specialization matrix approach:

- It employs a $2^{|A|} \times 2^{|A|}$-sized stochastic matrix $\mathbf{S}_A$. The computational and space complexity of the specialization matrix multiplication is $O(2^{|A|} \times 2^{|A|})$, a prohibitive burden even for modest FoD sizes.

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**References**

